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ANALYSIS OF SOFTWARE-LEVEL OPTIMIZATION SOLUTIONS FOR MOBILE WALKING RESCUE ROBOT LOCOMOTION CONTROL SYSTEM

This article will consider potential software-level options for the effective optimization of the four-legged walking robot locomotion control system by implementing optimization criteria parameters with further synthesis of regulators. A comparison of the synthesized control systems with optimal regulators will be presented to identify the best solutions that can be practically implemented to enhance the movement efficiency of the quadruped walking robot. The article will commence with the presentation of the dynamic mathematical model of the servo motor and its subsequent control system synthesis in order to create a sustainable foundation for further analysis of optimization solutions for the control system. The presented servomotor dynamics model will include a detailed derivation of expressions to form the resulting transfer function. Alongside the synthesis of the servomotor's dynamic model, a standard control system for the respective device, based on the obtained transfer function, will be introduced. Then, the synthesized control system will be analyzed from the perspective of optimality. As the next step, the before-presented mathematical model will be converted into the state-space representation form and the optimization criteria will be conducted, which is essential for the synthesis of an optimized solution. Following this, the synthesis of two potential optimization solutions in the form of control system regulators will be introduced. Among the synthesized regulators, two types of optimal regulators will be presented: a linear optimal regulator and a linear-quadratic optimal regulator with an integral component. In the concluding section, a comparative analysis of the obtained results will be conducted, and conclusions will be drawn based on the previous findings.

Key words: *control system, walker, evacuation, optimization, software.*

Introduction. Each year, due to climate change, wars, and humankind population expansion, we increasingly encounter emergencies where terrain becomes the prime obstacle to rescuing people [1, 2, 3]. As a result, standard evacuation means (for example, wheel-based platforms), due to their incapability of traversing complex and uneven terrain, have reduced efficiency under the aforementioned circumstances. Consequently, there is a growing need for mechanisms that can simplify or facilitate evacuation activities when transporting cargo or injured individuals over uneven terrain. One possible solution to this problem is the use of walking robots, which, unlike wheeled devices, are able to traverse complex terrain with extraordinary efficiency by utilizing legs. At the same time, their main disadvantage is speed [4, 6, 7]. To address this issue the solution to enhance locomotion speed by applying software-level optimization to the movement control system is needed.

Analysis of recent research and publications. First and foremost, we shall define the field of analysis. The prime idea of the article is to come up with an effective software-level solution for conventional servomotors in order to enhance their capabilities without any physical modification. Therefore, in the following paragraph, the conventional approach (standard servo motor control methods and their efficiency) shall be analyzed. In such a way we will define their efficiency, from the perspective of time, and enable a substantial basis for further optimization-focused analysis.

Let us begin by deriving the dynamic mathematical model for the servo motor. This step will provide an understanding of the basic principles for unoptimized servomotor control. First and foremost, we shall examine the physical processes occurring in the servomotor (fig. 1) [5, 7, 8, 9, 10, 11].

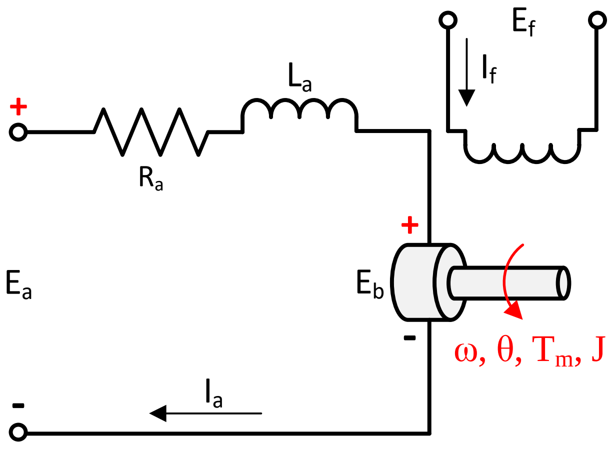

Fig. 1. Electrical diagram of the servomotor [8]

When a control constant voltage E_a is applied to the armature of the servo motor, it rotates and generates a back electromotive force E_b, which is proportional to the motor's speed [8, 9, 10, 11]:

$$E_b = k_a \cdot \theta' = k_a \cdot \omega, \quad (1)$$

where k_a is the proportionality coefficient of the electric field of the servomotor armature.

The differential equation that describes the electrical component of the servomotor will have the form:

$$E_a = R_a \cdot I_a + L_a \frac{dI_a}{dt} + E_b, \quad (2)$$

where R_a is the armature resistance; I_a is the current in the motor circuit; L_a s the armature inductance.

The torque T_m, generated by the servo motor is described by the following equation [8, 9, 10, 11]:

$$T_m = k_e \cdot I_a, \quad (3)$$

where k_e is the encoder (positioning sensor) coefficient. Although it is typically provided by the manufacturer, its value can be calculated using the following expression [8]:

$$k_e = \frac{\theta_{max}}{\gamma}, \quad (4)$$

where θ_{max} is the maximum range of the servo motor's rotation angle (working angle); γ s the resolution of the positioning sensor.

Now we shall write an alternative form of the equation (3) [8, 9, 10, 11]:

$$T_m = T + J \cdot \frac{d\omega}{dt} + B_0 \cdot \omega, \quad (5)$$

where J is the moment of inertia of the armature; B_0 is the friction coefficient; T is the engine shaft torque.

Let us perform the Laplace transformation for equations (1), (2), and (3):

$$E_b(s) = k_a \cdot \omega(s); \quad (6)$$

$$E_a(s) - E_b(s) = I_a(s) \cdot (s \cdot L_a + R_a); \quad (7)$$

$$T_m(s) = k_e \cdot I_a(s). \quad (8)$$

Substituting the function (6) into (7) we can find the equation for $I_a(s)$:

$$I_a(s) = \frac{E_a(s) - k_a \cdot \omega(s)}{s \cdot L_a + R_a}. \quad (9)$$

The next step is to perform the Laplace transformation for equation (15):

$$T_m(s) - T(s) = \omega(s) \cdot (s \cdot J + B_0). \quad (10)$$

From equation (10), by substituting equation (8) and performing transformations, we need to obtain the function for $\omega(s)$:

$$\omega(s) = \frac{k_e \cdot I_a(s) - T(s)}{s \cdot J + B_0}. \quad (11)$$

Let us generalize the transfer functions that characterize the processes in the servomotor along the channel «$E_a \to \omega$»:

$$W_1(s) = \frac{I_a(s)}{E_a(s) - E_b(s)} = \frac{1}{s \cdot L_a + R_a}; \quad (12)$$

$$W_2(s) = \frac{T_m(s)}{I_a(s)} = k_e; \quad (13)$$

$$W_3(s) = \frac{\omega(s)}{T_m(s) - T(s)} = \frac{1}{s \cdot J + B_0}; \quad (14)$$

$$W_4(s) = \frac{E_b(s)}{\omega(s)} = k_a. \quad (15)$$

Now we have all the necessary transfer functions and can obtain the equation for prime transfer function [8, 9, 10, 11]:

$$W(s) = \frac{W_1(s) \cdot W_2(s) \cdot W_3(s)}{1 + W_1(s) \cdot W_2(s) \cdot W_3(s) \cdot W_4(s)} = \frac{k_e}{(s \cdot L_a + R_a) \cdot (s \cdot J + B_0) + k_e \cdot k_a}. \quad (16)$$

Let us write the numerical values of all constants (Table 1). The presented values are relevant to servomotor at our disposal. Therefore, the values in the table may vary depending on which servomotor (or any other electric motor) is utilized. Despite this, the end result figure, in most cases, would maintain a similar shape and relevance.

In the following, by utilizing MATLAB Simulink software environment we will conduct research on the transfer function of the servomotor. For this reason, we will create the corresponding diagram in the abovementioned digital environment (fig. 2).

The diagram includes a previously undefined constant k. This is the conversion coefficient for angular velocity to the SI unit system (revolutions per minute). It is defined as follows [8]:

Table 1

Constants of the servomotor dynamic mode [8]

#	Parameter Name	Symbol	Unit of Measurement	Numerical Value
1	Armature resistance	R_a	Ohms	0,667
2	Armature inductance	L_a	Henries	$0,425 \cdot 10^{-3}$
3	Proportionality coefficient of the electric field	k_a	–	0,3
4	Control voltage	E_a	Volts	8,4
5	Encoder coefficient	k_e	–	0,293
6	Shaft torque of the servomotor	T	kg·cm	2,3
7	Friction coefficient	B	–	0,2
8	Moment of inertia	J	kg·m²	0,01

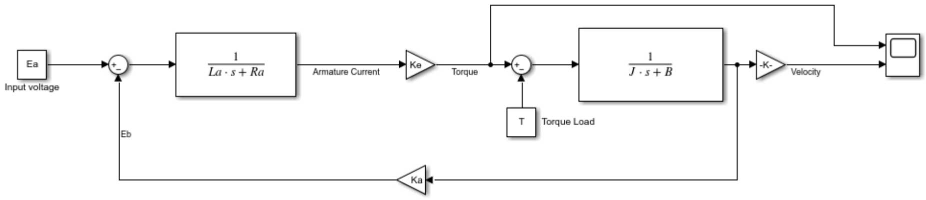

Fig. 2. Servomotor transfer function diagram in the MATLAB Simulink [8]

$$k = \frac{60}{2\pi}. \quad (17)$$

In addition, there is a disturbance T which is the servomotor shaft torque value.

Now we will conduct the transient characteristics for the channel $E_a \to \omega$ (fig. 3).

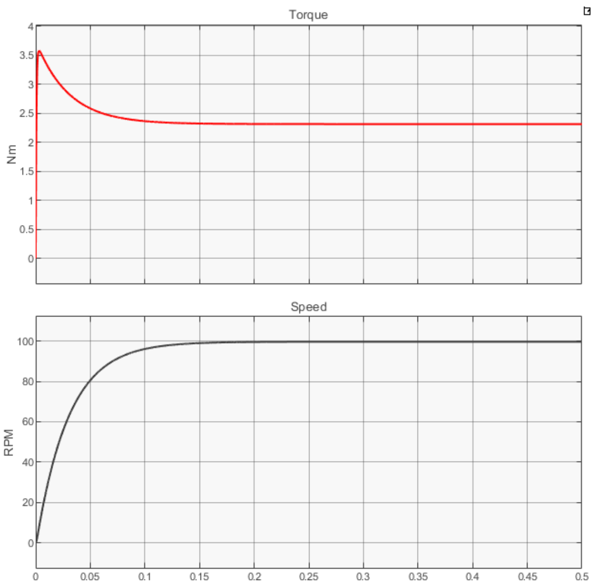

Fig. 3. Transient characteristics of the servo motor [8]

As shown in Figure 3 and in the transformations to obtain the overall transfer function, the value for the torque T can also be obtained from the general function in Figure 2 by placing the output after the proportional link k_a [8, 9, 10, 11].

In Figure 3 it can be seen that the transient process progresses smoothly, and the transport delay is about 0.187 seconds, which is acceptable but not an ideal result for the servo motor [8, 9]. For this reason, the optimization of the control system to minimize the transport delay is required.

The objectives of the article. Synthesize optimization solution for the walking robot servomotor (locomotion) control system to enhance system response time and, therefore, movement capabilities from the perspective of the speed parameter.

Such approach not only will enhance the movement capabilities of any walking robot, but will also enable cost-effective solution which, potentially, can severely reduce the manufacturing cost.

Solution. To solve this problem, we will select an integral quality criterion. According to the conditions stated previously, we include in the optimality criterion the servomotor speed and the shaft torque generated along with the speed:

$$I = \frac{1}{2}\int_0^{t_f} \left(q_{11}\left(\omega - \omega^{task}\right)^2 + q_{22}\left(T - T^{task}\right)^2 + rE_a^2 \right) dt \to min. \quad (18)$$

First, we shall bring the mathematical model to a form that corresponds to the standard representation for a system in state-space.

$$\begin{cases} \dfrac{d\left(x_1\right)}{dt} = a_{11}x_1 + a_{12}x_2 + b_1u_1 \\ \dfrac{d\left(x_2\right)}{dt} = a_{21}x_1 + a_{22}x_2 + b_2u_2 \end{cases} \quad (19)$$

For this transformation, we will use MATLAB software and built-in transition functions for state-space transformation. But first, we will rewrite the system's transfer function in the form that will be used after the transformation (20).

$$W(s)=\frac{k_e}{s^2 L_a J+s\left(B_0 L_a+R_a J\right)+\left(R_a B_0+k_e k_a\right)} \qquad (20)$$

Since the topic of this article is the study of optimization methods, the transition to the state space, as mentioned earlier, will be performed using MATLAB software, specifically with the built-in function [A, B, C, D] = ssdata(W) where W is the transfer function of our system, and A, B, C, D are the matrix elements of our transfer function in the state-space. The calculation result is shown in Figure 4.

```
A = 2×2
10^3 ×
    -1.5894   -0.2034
     0.2560         0

B = 2×1
     16
      0

C = 1×2
          0   16.8313

D = 0
```

Fig. 4. Mathematical model converted to the state-space

Thus, the system (1.19) in state space takes the form:

$$\begin{cases} \dfrac{d(x_1)}{dt}=a_{11}x_1+a_{12}x_2+b_1u_1 \\ \dfrac{d(x_2)}{dt}=a_{21}x_1 \end{cases} \qquad (21)$$

For the prime objective, we shall introduce the optimal linear regulator:

$$U(t)=-K(t)X(t); \qquad (22)$$

$$K(t)=-R^{-1}BP(t), \text{ де} \qquad (23)$$

The given equations define a linear optimal regulator (LOR) with a matrix gain coefficient *K*. This regulator minimizes the criterion along the system's trajectories, whereby:

1. the matrix gain coefficient K can be determined outside the control loop, as it does not depend on either *X* or *U*. To determine K, it is necessary to solve the Riccati equation in reverse time;
2. With constant matrices *A, B, R, Q* and as $t\to\infty$, P approaches a steady-state value, which can be found by solving the algebraic nonlinear matrix equation:

$$A^T P+PA-PBR^{-1}B^T P+Q=0, \qquad (24)$$

which, in turn, ensures the constancy of the matrix gain coefficient K of the regulator.

We shall introduce the necessary matrices and solve the Riccati equation:

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 0 \end{bmatrix}; B=\begin{bmatrix} b_{11} \\ 0 \end{bmatrix}; P=\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}. \qquad (25)$$

To avoid unnecessary calculations and transformations, we will obtain the values of the Riccati equation coefficients in MATLAB software (Figure 5).

```
R = 1
P = 2×2
        0.0007     0.0025
        0.0025     0.0158
```

Fig. 5. Solution for the Riccati Equation

Let us proceed to the creation of an optimal control system that includes a linear-quadratic regulator with an integral component (LQIR). We will directly introduce the derivative of the control signal into the quality criterion:

$$J=\frac{1}{2}X\left(t_f\right)^T SX\left(t_f\right)+\frac{1}{2}\int_0^{t_f}\left[X^T QX+\left(\frac{dU}{dt}\right)^T R\frac{dU}{dt}+2X^T N\frac{dU}{dt}\right]dt. \qquad (26)$$

In addition, we differentiate the dynamics equation:

$$\frac{d^2X}{dt^2}=A\frac{dX}{dt}+B\frac{dU}{dt}. \qquad (27)$$

Variable substitution:

$$V=\frac{dU}{dt}; W_1=X; W_2=\frac{dX}{dt}; W=[W_1 \; \vdots \; W_2]^T. \qquad (28)$$

Finally, we proceed to the equation:

$$\frac{dW}{dt}=A_1W+B_1V, \qquad (29)$$

where

$$A_1=\begin{bmatrix} 0 & \vdots & I \\ \cdots & \vdots & \cdots \\ 0 & \vdots & A \end{bmatrix}; B_1=\begin{bmatrix} 0 \\ \cdots \\ B \end{bmatrix}. \qquad (30)$$

Additionally, we shall make substitution in the quality criterion:

$$J=\frac{1}{2}\left[W^T S_1 W\right]_{t=t_f}=+\frac{1}{2}\int_0^{t_f}\left[W^T Q_1 W+V^T RV+2W^T NV\right]dt, \qquad (31)$$

where

$$Q_1=\begin{bmatrix} Q & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & 0 \end{bmatrix}; S_1=\begin{bmatrix} S & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & 0 \end{bmatrix}. \qquad (32)$$

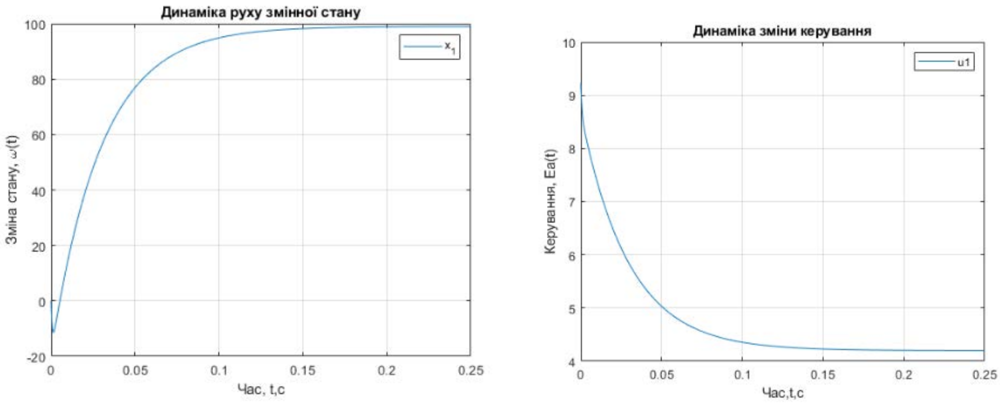

Fig. 5. Result of the LOR Synthesis

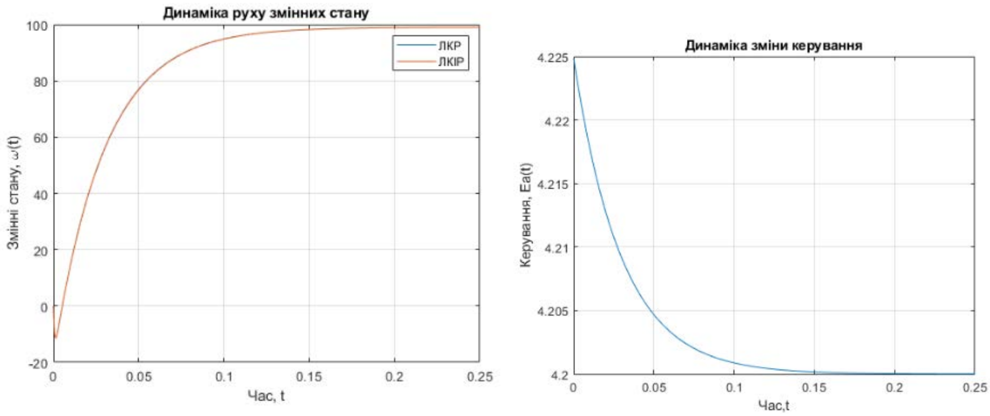

Fig. 6. Result of the LQRI Synthesis and its comparison with the LOR

The expression for optimal control will be as follows:

$$V = -KW = -\left[K_1 \;\vdots\; K_2\right]\begin{bmatrix} W_1 \\ \cdots \\ W_2 \end{bmatrix}. \qquad (33)$$

Rewriting using the original variables:

$$U = -K_2 X - \int_0^{\infty} \left(K_1 - \frac{dK_2}{dt}\right) X dt, \qquad (34)$$

we conducting the LQIR regulator.

Taking into account the above-presented equations and information we shall create LOR and LQRI regulators for our system in a digital environment (MATLAB software) and analyze their end effect on the response time. The results are presented in the Figures 5 and 6.

Conclusion. The data presented in Figures 5 and 6 indicates the insignificant difference between the prime result and the optimized one. Further analyzing the provided data, we can clearly indicate the absence of deviation between the two studied regulators. Finally, we incline that not only does any significant reason for the utilization of the more complex LQRI regulator exist but also the ultimate unnecessity of the regulator in general maintains its occurrence.

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Сазонов А.Ю., Ладієва Л.Р., Чередниченко В.І. АНАЛІЗ ПРОГРАМНИХ РІШЕНЬ ДЛЯ ОПТИМІЗАЦІЇ СИСТЕМИ КЕРУВАННЯ РУХОМ МОБІЛЬНОГО ЕВАКУАЦІЙНОГО РОБОТА

У представленій статті нами буде розглянуто потенційні варіанти ефективної оптимізації, на рівні програмного забезпечення, системи керування рухом чотириногого крокуючого робота за допомогою введення параметрів-критеріїв оптимізації з подальшим синтезом системи керування і відповідних регуляторів. Буде представлене порівняння синтезованих систем керування з оптимальними регуляторами з метою знаходження оптимальних рішень, які можуть бути застосовані у практичному середовищі для покращення ефективності пересування чотириногого крокуючого робота. Стаття розпочнеться з представлення динамічної математичної моделі сервомотору та його системи керування, щоб створити надійний фундамент для подальшого синтезу та аналізу оптимізаційних рішень для системи керування. Представлена модель динаміки серводвигуна міститиме детальне виведення виразів для формування результуючої передатної функції. Окрім виведення динамічної моделі серводвигуна, буде представлено стандартну систему керування відповідним пристроєм, сформовану на основі отриманої передатної функції. Синтезована система керування буде досліджена з точки зору оптимальності. На наступному етапі представлена математична модель буде перетворена у форму простору станів, що є необхідним кроком для синтезу оптимальних регуляторів. Даний етап потрібний для проведення процедури введення критеріїв оптимальності, необхідних для подальшого синтезу оптимізованого рішення. Далі буде представлено синтез двох потенційних рішень оптимізації у вигляді регуляторів системи керування. Серед синтезованих регуляторів буде представлено два види оптимальних регуляторів: лінійний оптимальний регулятор та лінійно-квадратичний оптимальний регулятор з інтегральною складовою. У заключній частині статті нами буде проведено порівняльний аналіз отриманих результатів та створено висновок на їх основі.

Ключові слова: *система керування, крокуючий робот, евакуаційний, оптимізація, програмне забезпечення.*